$$e_2^{(N+1)} = \frac{1.05 + e_3^{(N)}}{2} + \frac{0.05 f_2^{(N)}}{\left(e_2^{(N)}\right)^2 + \left(f_2^{(N)}\right)^2} \quad \text{(A)}$$

$$f_2^{(N+1)} = \frac{f_3^{(N)}}{2} - \frac{0.05\, e_2^{(N+1)}}{\left(e_2^{(N)}\right)^2 + \left(f_2^{(N)}\right)^2} \quad \text{(B)}$$

$$Q_3 = 10\left\{1 - e_3^{(N)} e_2^{(N+1)} - f_3^{(N)} f_2^{(N+1)}\right\} \quad \text{(C)}$$

$$e_3^{(N+1)} = e_2^{(N+1)} + \frac{0.1\, Q_3 e_3^{(N)} - 0.06 f_3^{(N)}}{\left(e_3^{(N)}\right)^2 + \left(f_3^{(N)}\right)^2} \quad \text{(D)}$$

correcting for magnitude we have

$$e_3^{(N+1)} = \frac{e_3^{(N+1)}\left[\left(e_3^{(N)}\right)^2 + \left(f_3^{(N)}\right)^2\right]^{1/2}}{\left[\left(e_3^{(N+1)}\right)^2 + \left(f_3^{(N+1)}\right)^2\right]^{1/2}}$$

$$f_3^{(N+1)} = \frac{f_3^{(N+1)}\left[\left(e_3^{(N)}\right)^2 + \left(f_3^{(N)}\right)^2\right]^{1/2}}{\left[\left(e_3^{(N+1)}\right)^2 + \left(f_3^{(N+1)}\right)^2\right]^{1/2}}$$

using P & Q mismatches as convergence criterion, convergence was achieved in 11 iterations to desired accuracy.

Results

Bus #	E_{pu}	F_{pu}	P_{pu}	Q_{pu}
1	1.05	0	4.00083×10^{-1}	2.8247×10^{-1}
2	1.02307	-3.8103×10^{-3}	-1.00000	-1.0769×10^{-6}
3	0.99977	2.14119×10^{-2}	6.00002×10^{-1}	-2.20166×10^{-1}

or

Bus #	$\lvert V \rvert_{pu}$	$\delta_{deg.}$	P_{pu}	Q_{pu}
1	1.05	0.0	Same as above.	
2	1.023017	-2.133		
3	0.9999926	1.227		

NEWTON-RAPHSON METHOD

Consider a single value function which satisfies

$$f(x) = 0 \qquad (14)$$

At some solution point x, the function is zero. At some initial guess $x^{(0)}$, then we take the taylor series expansion of the function; and we get.

$$f(x) = f(x^{(0)}) + \left.\frac{df}{dx}\right|_{x^{(0)}} \cdot \Delta x^{(0)} + \text{higher order terms.} \qquad (15)$$

where

$$\Delta x^{(0)} = x^{(1)} - x^{(0)}$$

Neglecting higher order terms, eqn 15 reduces to

$$f(x) = 0 = f(\overset{(0)}{x}) + \left.\frac{df}{dx}\right|_{\overset{(0)}{x}} \overset{(0)}{\Delta x}$$

then

$$\overset{(0)}{\Delta x} = \frac{-f(\overset{(0)}{x})}{\left.\frac{df}{dx}\right|_{\overset{(0)}{x}}} \qquad (17)$$

the iterative solution of equation 17, starting with initial guess $\overset{(0)}{x}$ will home in on the solution very quickly as the error is proportional to

$$(x - \overset{(0)}{x})^2$$

The update

$$\overset{(1)}{x} = \overset{(0)}{x} + \overset{(0)}{\Delta x}$$

In general

$$\overset{(N+1)}{x} = \overset{(N)}{x} + \overset{(N)}{\Delta x}$$

EX: find $\sqrt{2}$

SOL:

$$f(x) = 0 = (x^2 - 2) = 0$$

initial guess of $\overset{(0)}{x} = 1$

Iteration # 1

$$\overset{(0)}{\Delta x} = \frac{-(1-2)}{2x\Big|_{\overset{(0)}{x}}} = \frac{-(-1)}{2} = \frac{1}{2}$$

$$\overset{(1)}{x} = \overset{(0)}{x} + \Delta x = 1 + \tfrac{1}{2} = 1.5$$

Iteration #2

$$\overset{(1)}{\Delta x} = \frac{-(1.5^2-2)}{2(1.5)} = -0.0833$$

$$\overset{(2)}{x} = \overset{(1)}{x} + \Delta x = 1.5 - 0.0833 = 1.4166667$$

THE SINGLE VALUE APPROACH

can be extended for a vector valued function in $\underline{x}$

$$\underline{x}^T = [x_1, x_2, x_3 \ldots x_n]$$

starting with an initial guess vector $\overset{(0)}{\underline{x}}$

Let us extend first the above single valued function, to a function of 2 variables.

$$\left.\begin{array}{l} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{array}\right\} \underline{f} = \begin{bmatrix} f_1(\underline{x}) \\ f_2(x) \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Let us expand the functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ using the taylor series exponsition about an initial guess. vector

$$\overset{(0)}{\underline{X}} = \begin{bmatrix} \overset{(0)}{x_1} \\ \overset{(0)}{x_2} \end{bmatrix}$$

$$f_1(x_1, x_2) = f_1(x_1^{(0)}, x_2^{(0)}) + \left.\frac{\partial f_1}{\partial x_1}\right|_{\underline{x}^{(0)}} \Delta x_1^{(0)} + \left.\frac{\partial f_1}{\partial x_2}\right|_{\underline{x}^{(0)}} \Delta x_2$$

$$f_2(x_1, x_2) = f_2(x_1^{(0)}, x_2^{(0)}) + \left.\frac{\partial f_2}{\partial x_1}\right|_{\underline{x}^{(0)}} \Delta x_1 + \left.\frac{\partial f_2}{\partial x_2}\right|_{\underline{x}^{(0)}} \Delta x_2$$

now we will want to write this in higher order terms. And expanding to n variables in x, in matrix form, can be written as,

$$\underbrace{\boxed{\underbrace{\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}}_{\text{specified vector}} - \underbrace{\begin{bmatrix} f_1(x_1^{(0)}, x_2, \dots x_n) \\ f_2 \\ \vdots \\ f_n \end{bmatrix}}_{\text{calculated vector}}}}_{\text{termed } \Delta \underline{f}(\underline{x}^{(0)})} = \underbrace{\left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}\right|_{\underline{x}^{(0)}}}_{\text{Jacobian}} \underbrace{\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix}}_{\text{increment vector.}} \qquad (18)$$

$$\Delta \underline{f}(\underline{x}^{(0)}) = \left.\underline{J}\right|_{\underline{x}^{(0)}} \Delta \underline{x}^{(0)} \qquad (19)$$

$$\Delta \underline{x}^{(0)} = \left.\underline{J}^{-1}\right|_{\underline{x}^{(0)}} \Delta \underline{f}(\underline{x}^{(0)}) \qquad (20)$$

and the updated iteration schem is given by:

$$\underline{x}^{(N+1)} = \underline{x}^{(N)} + \Delta \underline{x}^{(N)}$$

$x^{(N+1)}$ is the updated vetor that hopefully makes $\Delta f \to 0$ (mismatches)

NEWTON-RAPHSON LOAD FLOW

We shall use the polar form of the load flow equations. The rectangular form can be easily developed. The polar form may be slightly better with regards to convergence. The choice is a matter of prefrence